Integration of them over y leads to

$$\begin{cases} \delta(z) = \beta\, e^{b\alpha z/a}\,, \\ \omega(z) = b\, a^2\, z + \gamma\,, \end{cases} \tag{14}$$

where β and γ are constants. We thus obtain the result

$$u(x,y,z,t) = F - a \cdot \tanh \cdot \left[\frac{\beta\, e^{\alpha(t+bz/a)} + a\,x + b\,y + a^2\, b\,z + \gamma}{2} \right]. \tag{15}$$

Sample 2: Solitary waves are a special case of the family, since we are able to assume that

$$\phi(y, z) + \lambda(z, t) = b\,y + c\,z + p\,t + h\,, \tag{16}$$

where b, c, p and h are constants. Constraint (10) then leads to

$$u(x,y,z,t) = F - a \cdot \tanh \cdot \left[\frac{a\,x + b\,y + c\,z + a\left(\frac{c}{b} - a^2\right) t + h}{2} \right]. \tag{17}$$

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[1] P. Clarkson and E. Mansfield, Nonlinearity, **7,** 975 (1994).
[2] M. Jimbo and T. Miwa, Publ. Res. Inst. Math. Sci., Kyoto Univ. **19,** 943 (1983).
[3] B. Dorizzi, B. Gammaticos, A. Ramani, and P. Winternitz, J. Math. Phys. **27,** 2848 (1986).
[4] B. Tian and Y. T. Gao, Z. Naturforsch. **51 a,** 171 (1996).
[5] Y. T. Gao and B. Tian, Computers Math. Applic. **30,** 97 (1995).
[6] B. Tian and Y. T. Gao, J. Phys. A **29,** 2895 (1996).
[7] W.-H. Steeb, Continuous Symmetrics, Lie Algebras, Differential Equations and Computer Algebra, World Scientific, Singapore 1996.

New Exact Solutions for a Generalized Breaking Soliton Equation

Yi-Tian Gao[a,b], Bo Tian[a], and Woopyo Hong[b]

[a] Depts. of Applied Physics and Mathematics, Beijing University of Aeronautics and Astronautics, Beijing 100083, China*
[b] Dept. of Physics, Catholic University of Taegu-Hyosung, Hayang, Kyongsan, Kyungbuk 712-702, South Korea

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The breaking soliton equations are a class of nonlinear evolution equations of broad interest in physical and mathematical sciences. In this paper, the application of the generalized tanh method with symbolic computation leads to new exact solutions for a generalized breaking soliton equation, of which the previously-obtained solutions are the special cases.

Within a decade, a class of nonlinear evolution equations, called the breaking soliton equations, has become a widely interesting subject in physical and mathematical sciences, as seen, e.g., in [1–5]. In addition to the rich mathematical properties, those equations are found to include the self-dual Yang-Mills equation, and to be of value in describing the (2+1)-dimensional interaction of the Riemann waves and long waves. A generalized breaking soliton equation [1, 2] reads as

$$(u_{xt} - 4\, u_x\, u_{xy} - 2\, u_y\, u_{xx} + u_{xxxy})_x = -\alpha^2\, u_{yyy}\,, \tag{1}$$

where α^2 is real. Two classes of exact solutions of (1) have been found, one of which is the solitary waves plus arbitrary functions of t [4], the other is linear with respect to y but independent of α^2 [5].

In this paper, we apply the computerized symbolic computation and generalized tanh method [6] to (1), assuming that the exact solutions are of the form

$$u(x, y, t) = \sum_{m=0}^{N} A_m(y, t) \cdot \tanh^m\left[\Psi(y, t)\, x + \Theta(y, t)\right], \tag{2}$$

Reprint request to Prof. Bo Tian;
* Mailing address.

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where the $A_m(y,t)$'s, $\Psi(y,t) \neq 0$ and $\Theta(y,t)$ are differentiable functions of y and t only, with $A_N(y,t) \neq 0$, while $N = 1$, determined by the leading-order analysis.

Inserting Ansatz (2) into (1) leads to a huge expression. Then we separate out the different terms. Equating to zero the coefficients of like powers of x, we obtain from the terms with x and x^3 the first constraint,

$$\Psi(y,t) = \psi = \text{constant}\,. \tag{3}$$

Equating to zero the coefficients of like powers of $\tanh(\Psi x + \Theta)$, we obtain more constraints on $\psi, A_1(y,t)$, $A_0(y,t)$ and $\Theta(y,t)$, one of which,

$$\tanh^6(\Psi x + \Theta)\colon\ [A_1(y,t) + 2\psi]\,\Theta_y(y,t) = 0\,, \tag{4}$$

leads to the following families of exact solutions of (1):

Family I:

With the condition that $A_1 = -2\,\psi$, we get the expressions for $\Theta(y,t)$ and $A_0(y,t)$ from the terms with $\tanh^3(\Psi x + \Theta)$ and $\tanh^4(\Psi x + \Theta)$, separately. The first family is thus found as

$$\begin{aligned} u^I(x,y,t) = &-2\,\psi \cdot \tanh\left[\psi\,x + \mu(t)\,y + \varpi(t)\right] \\ &+ \frac{y^2 \mu_t(t)}{4\,\psi} + \frac{y\,\varpi_t(t)}{2\,\psi} \\ &+ 2\,\psi\,y\,\mu(t) + \frac{\alpha^2\,y\,\mu^3(t)}{2\,\psi^3} + \gamma(t)\,, \end{aligned} \tag{5}$$

where the constant $\psi \neq 0$ as well as the differentiable functions $\mu(t)$, $\omega(t)$ and $\gamma(t)$ are all arbitrary.

The solutions obtained in [4, 5] are special cases of Family I.

Family II:

Starting from the assumption that $\Theta = \Theta(t)$ only, we obtain the expression for $A_0(y,t)$ from the terms with both $\tanh^2(\Psi x + \Theta)$ and $\tanh^4(\Psi x + \Theta)$, and get $A_1(y,t) = \tau =$ constant from the terms with $\tanh(\Psi x + \Theta)$, $\tanh^3(\Psi x + \Theta)$ and $\tanh^5(\Psi x + \Theta)$. The second family is calculated as

$$u^{II}(x,y,t) = \tau \cdot \tanh\left[\psi\,x + \Theta(t)\right] + \frac{y\,\Theta_t(t)}{2\,\psi} + \phi(t)\,, \tag{6}$$

where the constants $\psi \neq 0$ and $\tau \neq 0$ as well as the differentiable functions $\Theta(t)$ and $\phi(t)$ are all arbitrary. This family is independent of α.

The solutions obtained in [5] are a special case of Family II.

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[1] O. Bogoyavlenskii, Izv. Akad. Nauk SSSR Ser. Mat. **53,** 907 (1989).
[2] O. Bogoyavlenskii, Usp. Mat. Nauk **45,** 17 (1990).
[3] Y. Li, T. M. Φ. (a Russian journal) **99,** 441 (1994).
[4] Y. T. Gao and B. Tian, Computers Math. Applic. **33,** 35 (1996).
[5] Y. T. Gao and B. Tian, Chaos, Solitons and Fractals **8,** 879 (1997).
[6] B. Tian and Y. T. Gao, Mod. Phys. Lett. A **10,** 2937 (1995); Computer Phys. Comm. **95,** 139 (1996).

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